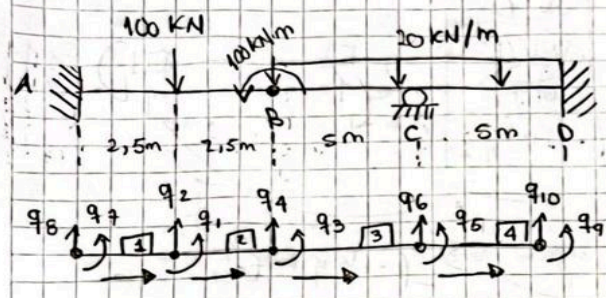
Sentido momento

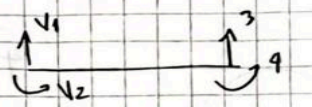
Estructura 2



Nodos = 5

Elementos = 4

GdL = 10 → libres: 5; Restr.: 5



Matriz de rigidez, fuerzas nodales (vector) y matriz de transformación

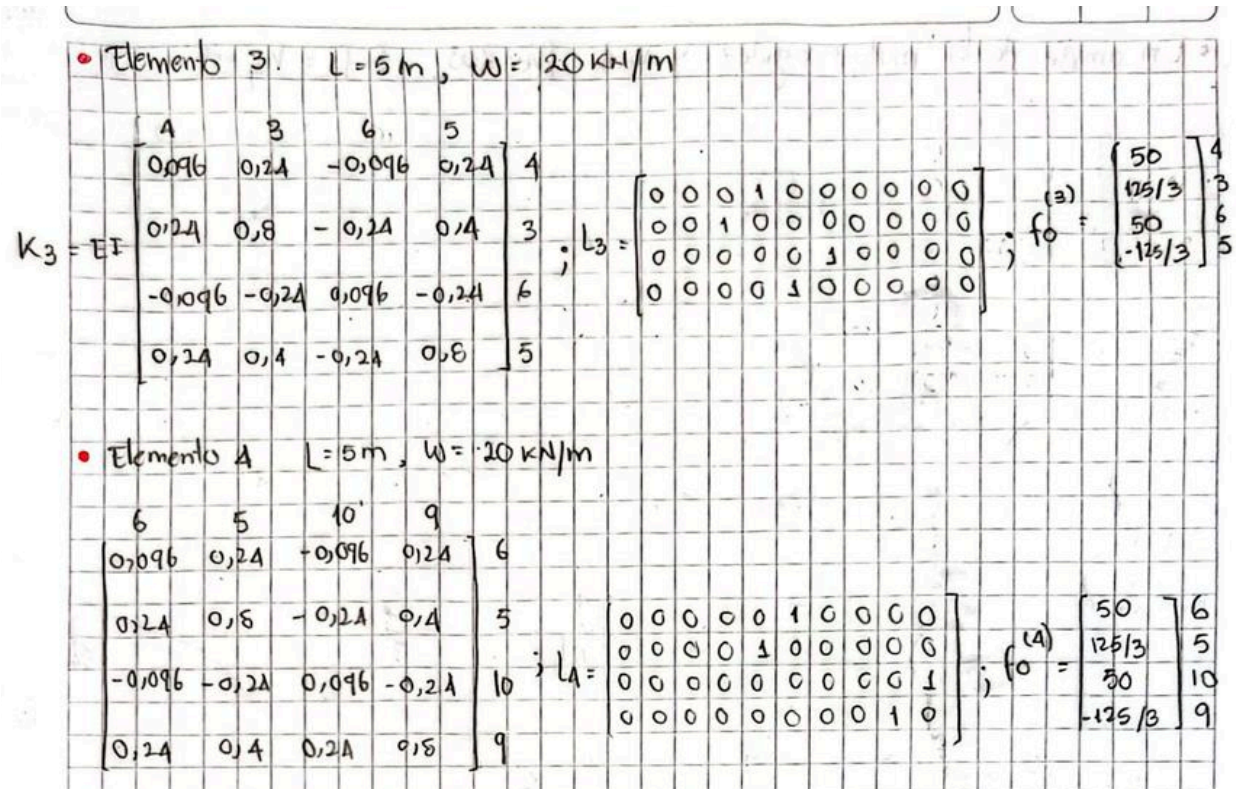
- Elemento 1. $L = 2{,}5\ m$, $w = 0$

$$k_1 = EI \begin{bmatrix} 8 & 7 & 2 & 1 \\ 0{,}768 & 0{,}96 & -0{,}768 & 0{,}96 \\ 0{,}96 & 1{,}6 & -0{,}96 & 0{,}8 \\ -0{,}768 & -0{,}96 & 0{,}768 & -0{,}96 \\ 0{,}96 & 0{,}8 & -0{,}96 & 1{,}6 \end{bmatrix} \begin{matrix} 8 \\ 7 \\ 2 \\ 1 \end{matrix} \qquad f_0^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 8 \\ 7 \\ 2 \\ 1 \end{matrix}$$

$$L_1 = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

- Elemento 2 $L = 2{,}5$, $w = 0$

$$k_2 = EI \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0{,}768 & 0{,}96 & -0{,}768 & 0{,}96 \\ 0{,}96 & 1{,}6 & -0{,}96 & 0{,}8 \\ -0{,}768 & -0{,}96 & 0{,}768 & -0{,}96 \\ 0{,}96 & 0{,}8 & -0{,}96 & 1{,}6 \end{bmatrix} \begin{matrix} 2 \\ 1 \\ 4 \\ 3 \end{matrix} ; \quad L_2 = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \end{bmatrix} ; \quad f_0^{(2)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 2 \\ 1 \\ 4 \\ 3 \end{matrix}$$



- Hallamos para cada elemento la matriz de rigidez en coordenadas globales, $K_q = L^T K_v L$

Elemento 1

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0.768 & 0.96 & -0.768 & 0.96 \\ 0.96 & 1.6 & -0.96 & 0.8 \\ -0.768 & -0.96 & -0.768 & -0.96 \\ 0.96 & 0.8 & -0.96 & 1.6 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_{q1} = 2EI \begin{pmatrix} 1.6 & -0.96 & 0 & 0 & 0 & 0 & 0.8 & 0.96 & 0 & 0 \\ -0.96 & -0.768 & 0 & 0 & 0 & 0 & -0.96 & -0.768 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & -0.96 & 0 & 0 & 0 & 0 & 1.6 & 0.96 & 0 & 0 \\ 0.96 & -0.768 & 0 & 0 & 0 & 0 & 0.96 & 0.768 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Elemento 2

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0.768 & 0.96 & -0.768 & 0.96 \\ 0.96 & 1.6 & -0.96 & 0.8 \\ -0.768 & -0.96 & -0.768 & -0.96 \\ 0.96 & 0.8 & -0.96 & 1.6 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_{q2} = 2EI \begin{pmatrix} 1.6 & 0.96 & 0.8 & -0.96 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.96 & 0.768 & 0.96 & -0.768 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.96 & 1.6 & -0.96 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.96 & -0.768 & -0.96 & -0.768 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Elemento 3

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.8 & -0.24 & 0.4 \\ -0.096 & -0.24 & -0.096 & -0.24 \\ 0.024 & 0.4 & -0.24 & 0.8 \end{pmatrix} .$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_{q3} = EI \cdot \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.8} & \mathbf{0.24} & \mathbf{0.4} & \mathbf{-0.24} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.24} & \mathbf{0.096} & \mathbf{0.24} & \mathbf{-0.096} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.4} & \mathbf{0.024} & \mathbf{0.8} & \mathbf{-0.24} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.24} & \mathbf{-0.096} & \mathbf{-0.24} & \mathbf{-0.096} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

Elemento 4

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.8 & -0.24 & 0.4 \\ -0.096 & -0.24 & -0.096 & -0.24 \\ 0.024 & 0.4 & -0.24 & 0.8 \end{pmatrix} .$$

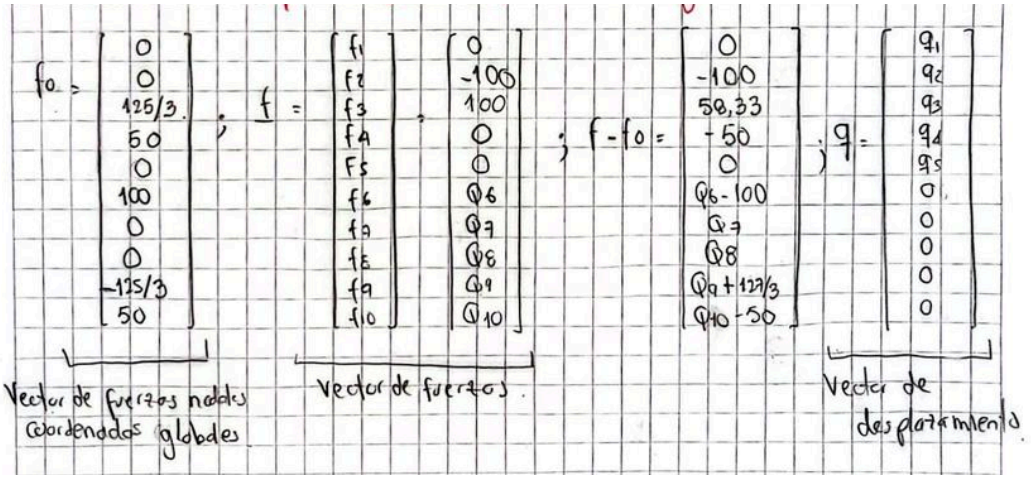
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$K_{q4} = EI \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.24 & 0 & 0 & 0.4 & -0.24 \\ 0 & 0 & 0 & 0 & 0.24 & 0.096 & 0 & 0 & 0.24 & -0.096 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.024 & 0 & 0 & 0.8 & -0.24 \\ 0 & 0 & 0 & 0 & -0.24 & -0.096 & 0 & 0 & -0.24 & -0.096 \end{pmatrix}$$

- Luego se calcula la matriz de rigidez total en coordenadas globales.
  $K_q = K_{q1} + K_{q2} + K_{q3} + K_{q4}$

$$K_q = EI \begin{pmatrix} 12.8 & 0 & 3.2 & -3.84 & 0 & 0 & 3.2 & 3.84 & 0 & 0 \\ 0 & 0 & 3.84 & -3.072 & 0 & 0 & -3.84 & -3.072 & 0 & 0 \\ 3.2 & 3.84 & 6.4 & -3.84 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3.84 & -3.072 & -3.84 & -3.072 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.2 & -3.84 & 0 & 0 & 0 & 0 & 6.4 & 3.84 & 0 & 0 \\ 3.84 & -3.072 & 0 & 0 & 0 & 0 & 3.84 & 3.072 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Se calculan los vectores de fuerzas nodales globales, vectores de fuerzas y de desplazamiento de la estructura.



- Se ensambla la matriz la matriz de rigidez, vector de fuerza y vector de desplazamiento. $f - f_0 = K_q * q$

$$\begin{pmatrix} 0 \\ -100 \\ 58.33 \\ -50 \\ 0 \\ Q6-100 \\ Q7 \\ Q8 \\ Q9+\frac{125}{3} \\ Q10-50 \end{pmatrix} = EI \left(\begin{array}{ccccc|ccccc} 6.4 & 0 & 1.6 & -1.92 & 0 & 0 & 1.6 & 1.92 & 0 & 0 \\ 0 & 0 & 1.92 & -1.536 & 0 & 0 & -1.92 & -1.536 & 0 & 0 \\ 1.6 & 1.92 & 4 & -1.68 & 0.4 & -0.24 & 0 & 0 & 0 & 0 \\ -1.92 & -1.536 & -1.68 & -1.44 & 0.24 & -0.096 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.024 & 1.6 & 0 & 0 & 0 & 0.4 & -0.24 \\ \hline 0 & 0 & -0.24 & -0.096 & 0 & 0 & 0 & 0 & 0.24 & -0.096 \\ 1.6 & -1.92 & 0 & 0 & 0 & 0 & 3.2 & 1.92 & 0 & 0 \\ 1.92 & -1.536 & 0 & 0 & 0 & 0 & 1.92 & 1.536 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.024 & 0 & 0 & 0.8 & -0.24 \\ 0 & 0 & 0 & 0 & -0.24 & -0.096 & 0 & 0 & -0.24 & -0.096 \end{array}\right) \begin{pmatrix} q1 \\ q2 \\ q3 \\ q4 \\ q5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

K11 K12 K21 K22